Dynamic permeability of electrically conducting fluids under magnetic fields in annular ducts

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(Received 14 December 2000; published 25 June 2001)

The dynamic response of an electrically conducting fluid (either Newtonian or Maxwellian) flowing between straight concentric circular cylinders under a constant radial magnetic field, is analyzed. The isothermal flow is studied using the time Fourier transform, so that the dynamic generalization of Darcy's law in the frequency domain is obtained and analytical expressions for the dynamic permeability are derived. For the Newtonian case, the range of frequencies where the dynamic permeability approaches the static value is enlarged the smaller the gap between the cylinders and the higher the magnetic-field strength. For the Maxwell fluid, the presence of the inner cylinder shifts the frequencies that lead to the enhancement of the real part of the dynamic permeability to larger values and increases its maximum values relative to the case where the inner cylinder is absent. In addition, the Ohmic dissipation causes the damping of the amplitude of the response.

DOI: 10.1103/PhysRevE.64.016313 PACS number(s): 47.60.+i, 47.50.+d, 47.65.+a

I. INTRODUCTION

Simultaneous transport of mass, momentum, and energy in electrically conducting fluids under electromagnetic fields is of great importance in many fields of science and technology. Several examples of this kind of transport can be found in a variety of duct geometries in nuclear reactors, electromagnetic pumps and flowmeters, and inkjet printers [1–4], among others. Whenever the transport involves frequency dependent processes, a suitable description can be carried out in terms of the dynamic permeability function. Although the study of this function has been the topic of many previous researches dealing with the flow of Newtonian as well as viscoelastic fluids in straight tubes [5–10], it appears that the influence of different tube geometries has not been widely investigated. This is not the case for steady-state problems where flow and heat transfer have been extensively studied in a variety of geometries, for instance, annular ducts [11]. Besides, the dynamic permeability of flows of electrically conducting fluids in tubes under electromagnetic fields has not received any attention either. Therefore, the objective of this paper is twofold. First, to explore the effect on the dynamic permeability of altering the common single tube geometry by introducing an inner solid cylinder, in such a way that the flow takes place in an annular duct, and second, to discern how the presence of a constant magnetic field affects the dynamic permeability in tube flows when the fluid is electrically conducting.

The understanding of how electromagnetic fields influence the dynamic permeability may be of relevance in physiological and metallurgical applications, particularly those related to the development of electromagnetic pumps and flowmeters, which are also widely used in the chemical and nuclear industries. In such devices, flows of conducting fluids under magnetic fields take place in frequency dependent pressure drops [12–14]. Some of these flows involve the motion of Newtonian fluids, but in many practical situations fluids are far from presenting a Newtonian behavior. That is the case of the flow of blood in electromagnetic flowmeters [13], which can be approximately modeled using a Maxwell fluid [15]. Also, in certain metallurgical processes, it is important to take into account that liquid metals at temperatures, slightly higher than the melting point, present a non-Newtonian behavior [16]. Likewise, the motion of multiphase dispersed conducting media fail to be described by the magnetohydrodynamics (MHD) of Newtonian fluids. Incidentally, many media may acquire or modify non-Newtonian properties by the application of strong magnetic fields, as is the case of magnetorheological (MR) fluids [17].

In this paper, we explore the dynamic permeability of fully developed flows of electrically conducting fluids, either Newtonian or Maxwellian, in a horizontal annular duct under a constant radial magnetic field. We are concerned, as in ordinary MHD, with the motion of nonferromagnetic media, and accordingly the magnetic permeability of the fluid is assumed to be that of the vacuum, $\mu=\mu_0$. The inductionless approximation is assumed [18], so that magnetic fields induced by the currents circulating in the fluid are neglected. These currents, however, interact with the magnetic field and produce a Lorentz force that alters the fluid motion. Also, they are an additional source of (Ohmic) dissipation in the fluid, which is characterized by a dissipative time scale (Joule time). With the aim at getting analytical expressions, the problem is transformed to the frequency domain through a time Fourier transform and a dynamic generalization of Darcy's law is obtained. Thus, analytic expressions for the frequency-dependent permeability for the mean flow of Newtonian and Maxwellian fluids are used to explore the dynamic behavior in a wide range of frequencies. Specific transient regimes as well as entrance effects are not considered in this paper. However, transient problems can be described using the inverse Fourier transform. The main objective here is to characterize the effects of both the annular gap of the duct and the magnetic-field strength on the dynamic permeability of conducting fluids in tubes. In fact, these parameters modify the flow structure through boundary conditions and an additional body force. Evidently, they also influence the dissipation within the flow. A particular

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interrogation to be answered is how the resonant behavior that leads to the enhancement of the dynamic permeability in Maxwell fluids in tubes [6–8] is affected by geometrical factors and the magnetic interaction.

The organization of the paper is as follows. In the Sec. II, the formulation of the problem is presented and the dynamic permeability of a Newtonian fluid in an annular duct under a radial magnetic field is calculated. Results for different annular gaps and magnetic field strengths are also discussed. In Sec. III, the calculation of the dynamic permeability in the former conditions is carried out but considering a viscoelastic fluid modeled through the linear Maxwell model. A parametric study considering different gaps, magnetic-field strengths, and elastic relaxation parameters is performed. Finally, in Sec. IV concluding remarks are stated.

II. DYNAMIC PERMEABILITY IN A NEWTONIAN FLUID

Let us consider the isothermal laminar flow of a Newtonian, electrically conducting incompressible fluid in the gap formed by two coaxial infinite cylinders. A transverse radial magnetic field is assumed to be imposed and the walls of the cylinders are assumed to be electrically insulated. The experimental conditions required to obtain a radial magnetic field in this geometry are discussed in Ref. [19]. The axial motion of the conducting fluid in the presence of the applied radial magnetic field, $\mathbf{B_0}$, induces an electric current density in the polar direction. In turn, this current generates an axial magnetic field $\mathbf{b}$, that can be understood as a perturbation produced by the fluid motion. The interaction of the current density with the total field, $\mathbf{B}=\mathbf{B_0}+\mathbf{b}$, originates a Lorentz force, namely, $\mathbf{j}\times\mathbf{B}$, where $\mathbf{j}$ is the electric current density. The radial component of the field gives rise to an axial force that opposes the fluid motion and, therefore, for a given flow rate the axial pressure gradient that drives the flow must be stronger than the one required in the absence of magnetic field. In turn, the interaction of the current and the axial induced field produces a radial (irrotational) force, which is balanced by a radial pressure gradient. Thus, from the incompressibility condition, the continuity equation reads

$$\nabla\cdot\mathbf{u}=0. \tag{1}$$

Since we consider a fully developed flow, the nonlinear convective term vanishes and the momentum balance equation reduces to

$$\rho\frac{\partial\mathbf{u}}{\partial t}=-\nabla p-\nabla\cdot\boldsymbol{\tau}+\mathbf{j}\times\mathbf{B}, \tag{2}$$

where ρ is the mass density of the fluid, $\mathbf{u}$ is the velocity field, p is the pressure field, and τ represents the viscous stress tensor. Notice that the linearized Eq. (2) corresponds to the approximation of low Reynolds number flows. The constitutive relations for $\boldsymbol{\tau}$ and $\mathbf{j}$ must also be supplied. For a Newtonian incompressible fluid we have

$$\boldsymbol{\tau}=-\eta\nabla\mathbf{u}, \tag{3}$$

where η is the dynamic viscosity of the fluid. In turn, Ohm's law establishes that

$$\mathbf{j}=\sigma(\mathbf{E}+\mathbf{u}\times\mathbf{B}), \tag{4}$$

where σ is the electrical conductivity of the fluid and $\mathbf{E}$ is the electric field. In addition, we consider the electromagnetic field equations in the quasistatic approximation, namely [20],

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \tag{5}$$

$$\nabla\times\mathbf{B}=\mu\mathbf{j}, \tag{6}$$

$$\nabla\cdot\mathbf{B}=0. \tag{7}$$

In principle, Eqs. (1)–(7) are coupled due to electromagnetic induction effects. Here we assume that the axial induced field $\mathbf{b}$ is much smaller than the applied field. This occurs provided the magnetic Reynolds number, $R_m=\mu\sigma UL$, is much less than unity, where μ is the magnetic permeability of the fluid, and U and L are characteristic velocity and length of the flow [18]. Hence, under the approximation $R_m\ll 1$, $\mathbf{b}$ can be neglected. This means that the magnetic field is unperturbed by the fluid motion and satisfies the magnetostatic equations [18], i.e.,

$$\nabla\cdot\mathbf{B}=0, \quad \nabla\times\mathbf{B}=0. \tag{8}$$

Here, we consider that the magnetic field is given by $\mathbf{B}=B_o\mathbf{e}_r$, where B_o is constant and $\mathbf{e}_r$ is the unit vector in the radial direction. Since the magnetic field is constant, Faraday's law of induction (5) leads to $\nabla\times\mathbf{E}=0$, and the electric field becomes potential, namely, $\mathbf{E}=-\nabla\phi$, where ϕ is the electrostatic potential. In the analyzed case, there are no external electric fields, therefore, $\mathbf{E}=0$, which means that the azimuthal current loops form perfect short circuits. Under these conditions Ohm's law reduces to

$$\mathbf{j}=\sigma\mathbf{u}\times\mathbf{B}. \tag{9}$$

Substituting Eqs. (3) and (9) in Eq. (2), and taking into account that the axial velocity component $v(r,t)$, is the only one, we find that the momentum balance equation becomes

$$\frac{\partial v}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\eta}{\rho}\nabla^2 v-\frac{\sigma B_o^2}{\rho}v. \tag{10}$$

We take the time Fourier transform of Eq. (10) and obtain

$$\nabla^2 V+\frac{\rho}{\eta}\left(i\omega-\frac{1}{t_j}\right)V=\frac{1}{\eta}\frac{\partial P}{\partial z}, \tag{11}$$

where V and P stand for the Fourier transform of v and p, respectively, and $t_j=\rho/\sigma B_o^2$ is the Joule time that characterizes the Ohmic dissipation in the fluid. The solution of Eq. (11) under nonslip boundary conditions is given by

$$V(r,\omega)=\frac{1}{\eta\beta_n^2}\left[1-\frac{J_o(\beta_n r)-C_n N_o(\beta_n r)}{J_o(\beta_n a)-C_n N_o(\beta_n a)}\right]\frac{\partial P}{\partial z}, \tag{12}$$

where $J_o(x)$ and $N_o(x)$ are the zeroth-order Bessel functions of the first and second kind, respectively, and

$$\beta_n=\left[\frac{\rho}{\eta}\left(i\omega-\frac{1}{t_j}\right)\right],\qquad C_n=\frac{J_o(\beta_n a)-J_o(\beta_n b)}{N_o(\beta_n a)-N_o(\beta_n b)},$$

with a and b being the radius of the outer and inner cylinders, respectively.

The average flow rate in the cross section of the annular channel is given by

$$Q(\omega)=2\pi\int_b^a V(r,\omega)r\,dr=-\frac{K_n(\omega)}{\eta}\frac{\partial P}{\partial z},\qquad (13)$$

where the frequency-dependent permeability $K_n(\omega)$ can be expressed as

$$K_n(\omega)=-\frac{\pi a^4}{\varpi_n}\left\{1-\frac{2[J_1(\sqrt{\varpi_n})-C_nN_1(\sqrt{\varpi_n})]}{\sqrt{\varpi_n}[J_o(\sqrt{\varpi_n})-C_nN_o(\sqrt{\varpi_n})]}-R^2\left(1-\frac{2[J_1(R\sqrt{\varpi_n})-C_nN_1(\sqrt{R\varpi_n})]}{R\sqrt{\varpi_n}[J_o(\sqrt{\varpi_n})-C_nN_o(\sqrt{\varpi_n})]}\right)\right\}.\qquad (14)$$

The dimensionless parameters appearing in Eq. (14) are given by

$$\varpi_n=i\omega_n^*-M^2,\quad \omega_n^*=\omega t_v,$$

$$t_v=\frac{\rho a^2}{\eta},\quad M^2=B_0^2a^2\frac{\sigma}{\eta}=\frac{t_v}{t_j},\quad R=\frac{b}{a}.$$

Notice that $\omega_n^*=\rho\omega a^2/\eta$ is the ratio of the characteristic time for viscous diffusion of momentum t_v and the time scale of the imposed pressure gradient $1/\omega$. ω_n^* may also be interpreted as a Reynolds number based on the characteristic velocity ωa. In turn, M is the Hartmann number that gives an estimate of the ratio of the characteristic time of viscous dissipation compared to the characteristic time of the magnetic damping due to electric currents circulating in the fluid. It is convenient to normalize the dynamic permeability using the static permeability $K_n(0)$, calculated for the steady flow, i.e.,

$$K_n^*(\omega_n^*)=\frac{K_n(\omega)}{K_n(0)},\qquad (15)$$

where of course

$$K_n(0)=\frac{\pi a^4}{M^2}\left\{1-\frac{2[J_1(iM)-C_0N_1(iM)]}{iM[J_0(iM)-C_0N_0(iM)]}-R^2\left(1-\frac{2[J_1(iRM)-C_0N_1(iRM)]}{iRM[J_0(iM)-C_0N_0(iM)]}\right)\right\},\qquad (16)$$

$$C_0=\frac{J_0(iM)-J_0(iRM)}{N_0(iM)-N_0(iRM)}.$$

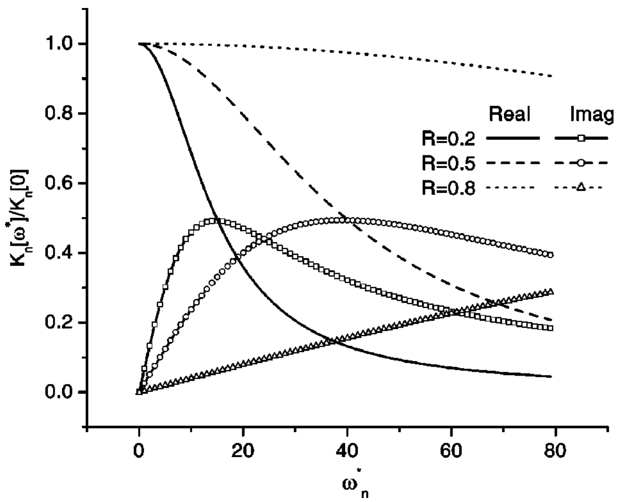

FIG. 1. Real and imaginary parts of the dimensionless dynamic permeability of a Newtonian fluid as a function of the normalized frequency parameter ω_n^* for $M=0$ and different values of the dimensionless gap R.

Let us first consider the flow in the absence of a magnetic field $(M=0)$. In that case, the dynamic permeability reduces to

$$\lim_{M\to 0}K_n^*(\omega_n^*)=-\frac{8}{i\omega_n^*(1-R^4+(1-R^2)^2/lnR)}\times\left\{1-\frac{2[J_1(\sqrt{i\omega_n^*})-C_nN_1(\sqrt{i\omega_n^*})]}{\sqrt{i\omega_n^*}[J_0(\sqrt{i\omega_n^*})-C_nN_0(\sqrt{i\omega_n^*})]}-R^2\left(1-\frac{2[J_1(R\sqrt{i\omega_n^*})-C_nN_1(R\sqrt{i\omega_n^*})]}{R\sqrt{i\omega_n^*}[J_0(\sqrt{i\omega_n^*})-C_nN_0(\sqrt{i\omega_n^*})]}\right)\right\},\qquad (17)$$

where for this limit

$$C_n=\frac{J_0(\sqrt{i\omega_n^*})-J_0(R\sqrt{i\omega_n^*})}{N_0(\sqrt{i\omega_n^*})-N_0(R\sqrt{i\omega_n^*})}.$$

In Fig. 1 the real and imaginary parts of the dynamic permeability as functions of the dimensionless frequency parameter ω_n^* are shown for $M=0$ and different values of the dimensionless gap R. For small values of R, corresponding to large gaps between the cylinders, the real part of the dynamic permeability takes the maximum value of 1 at zero frequency and decreases smoothly to zero as the ω_n^* increases. This is the typical relaxing viscous behavior observed in Newtonian flow in tubes. In fact, in the limit $R\to 0$, Eq. (17) reduces to

$$\lim_{M,R\to 0}K_n^*(\omega_n^*)=-\frac{8}{i\omega_n^*}\left\{1-\frac{2[J_1(\sqrt{i\omega_n^*})]}{\sqrt{i\omega_n^*}J_o(\sqrt{i\omega_n^*})}\right\},\quad (18)$$

which corresponds to the dynamic permeability obtained in Ref. [5] for the flow of a Newtonian fluid in a tube. From the imaginary part of the dynamic permeability, it can be observed that for small values of R there is a nonzero relative phase between the flow and the pressure gradient, except for very large frequencies. On the other hand, as the parameter R increases (i.e., the gap is reduced), the rate of decrease of the real part of $K_n^*(\omega)$ as the frequency grows is lessened. Actually, for very small gaps (e.g., $R=0.8$) the real part of the dynamic permeability, $\mathrm{Re}\{K_n^*(\omega)\}$, approaches 1 in a wide range of frequencies and decays to zero for very large frequencies. Therefore, as the gap decreases, two main effects are observed: the reduction of the dimensional dynamic and static permeabilities and the enlargement of the spectrum of perturbations that are able to pass through the fluid; likewise, the relative phase between the flow and the pressure gradient decreases. Evidently, in the limit $R\to 1$, $K_n(0)=K_n(\omega)=0$ and it can be shown from Eq. (17) that this occurs in such a way that $K_n^*(\omega_n^*)\to 1$.

It is illustrative to look at the different terms in Eq. (11), which establishes the balance among inertial, pressure, viscous, and magnetic forces in Fourier space. In dimensionless form, this equation can be expressed as

$$-i\omega_n^*\hat{V}=-M^2\frac{\partial\hat{P}}{\partial\hat{z}}+\hat{\boldsymbol{\nabla}}^2\hat{V}-M^2\hat{V}, \tag{19}$$

where $\hat{V}$ and $\hat{P}$ are normalized by a characteristic velocity and pressure, U and σB_o^2Ua, respectively. In turn, coordinates $\hat{r}$ and $\hat{z}$ are normalized by a. In the absence of a magnetic field ($M=0$), the balance is established among the inertial or local acceleration on the left-hand side of Eq. (19) and the pressure gradient and viscous forces on the right-hand side. Due to the inertial term, the velocity field will not generally be in phase with the pressure gradient. When the gap between the cylinders is reduced, viscous effects are increased as the boundary layers attached to each cylinder get closer. Therefore, for very small gaps, the acceleration term becomes negligible compared to the viscous term, which is only balanced by the pressure gradient, as in the static flow. Evidently, the acceleration term will also be negligible when $\omega_n^*\ll 1$. Notice that as long as the inertial term has some influence, the dynamic permeability will relax for some (large) frequency and a nonzero relative phase will exist. When $\omega_n^*\gg 1$, the flow behavior is more complicated. In that case, in the core of the flow, the acceleration term is dominant and is balanced by the pressure gradient, while near the walls of the cylinders, thin boundary layers are formed where viscous effects are not negligible. It can be shown from an asymptotic analysis[1] that for $\omega_n^*\gg 1$, the dynamic permeability decreases as $1/\omega_n^*$ in the core and as $e^{-\omega_n^*}/\omega_n^*$ in the boundary layers. Therefore, in the limit $\omega_n^*\to\infty$, $\mathrm{Re}\{K_n^*(\omega_n^*)\}\to 0$.

Let us turn our attention to the MHD case. Figure 2 shows the real and imaginary parts of the dynamic permeability for $R=0.1$ and Hartmann numbers 0.1, 10, and 100. For $M=0.1$ the magnetic interaction is negligible and the relaxing viscous behavior is again observed. In turn, for $M=10$, the relaxation of the dynamic permeability is less pronounced, while for $M=100$, $\mathrm{Re}\{K_n^*(\omega_n^*)\}$ approaches 1. In fact, it is observed from Eq. (14) that $K_n(\omega)$ reduces to $K(0)$ when $M^2\gg\omega_n^*$. In that case, the acceleration force in Eq. (19) becomes negligible, producing a quasisteady flow with well-differentiated regions: the core, where the magnetic force is balanced by the pressure gradient, and the boundary layers [Hartmann layers with a $O(M^{-1})$ thickness] where viscous effects are important. From the asymptotic expressions of Bessel functions for large values of the argument, it is found that for $M\gg 1$,

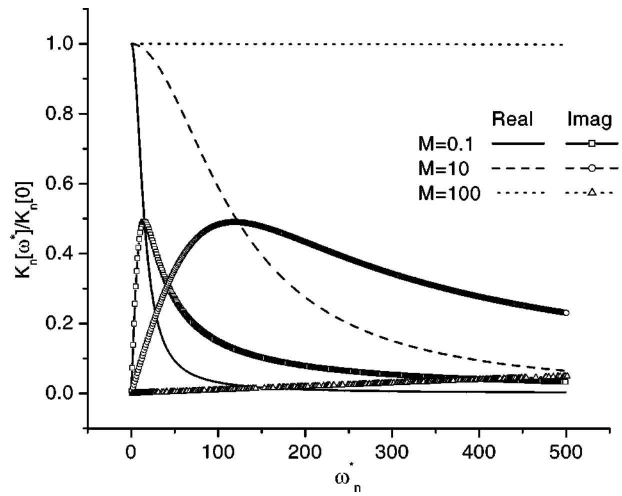

FIG. 2. Real and imaginary parts of the dimensionless dynamic permeability of a Newtonian fluid as a function of the normalized frequency parameter ω_n^* for $R=0.2$ and different values of the Hartmann number M.

$$K_n(\omega)=K(0)=\frac{\pi a^4}{M^2}(1-R^2), \tag{20}$$

which clearly vanishes as $M\to\infty$. Here it is important to stress that the condition $M\to\infty$ enlarges the frequency range where the dynamic permeability approaches the static value.

In the next section we turn our attention to the simplest model of a viscoelastic fluid.

III. DYNAMIC PERMEABILITY IN A MAXWELL FLUID

In order to analyze the dynamic permeability of a viscoelastic fluid under an imposed magnetic field, we use the linear form of the Maxwell fluid, namely,

$$t_m\frac{\partial\tau}{\partial t}=-\eta\boldsymbol{\nabla}\mathbf{u}-\tau, \tag{21}$$

where t_m is the Maxwell relaxation time. In the limit $t_m\to 0$, the Newtonian behavior (3) is recovered. Using Eqs. (2), (9), and (21), the equation of motion is obtained, namely,

[1]The flow in the limit case $R=0$ is treated in Ref. [21].

$$t_m \frac{\partial^2 v}{\partial t^2} + \frac{\partial v}{\partial t} = -\frac{1}{\rho}\left(1 + t_m \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \frac{\eta}{\rho}\nabla^2 v - \frac{\sigma B_o^2}{\rho}\left(1 + t_m \frac{\partial}{\partial t}\right) v. \tag{22}$$

Performing the time Fourier transform, we get

$$\nabla^2 V + \beta_m^2 V = \frac{(1 - i\omega t_m)}{\eta}\frac{\partial P}{\partial z}, \tag{23}$$

where

$$\beta_m = \left\{\frac{\rho}{\eta t_m}\left[(\omega t_m)^2 - \frac{t_m}{t_j} + i\omega t_m\left(1 + \frac{t_m}{t_j}\right)\right]\right\}^{1/2}.$$

The solution to Eq. (23) that satisfies nonslip boundary conditions at the walls of the cylinders is given by

$$V(r,\omega) = \frac{(1 - i\omega t_m)}{\eta \beta_m^2}\left[1 - \frac{J_0(\beta_m r) - C_m N_0(\beta_m r)}{J_0(\beta_m a) - C_m N_0(\beta_m a)}\right]\frac{\partial P}{\partial z}, \tag{24}$$

where

$$C_m = \frac{J_0(\beta_m a) - J_0(\beta_m b)}{N_0(\beta_m a) - N_0(\beta_m b)}.$$

Hence, calculating the average flow rate through Eq. (13), the dynamic permeability can be written as

$$K_m(\omega) = -\frac{\pi a^4 (1 - i\omega t_m)}{\alpha \varpi_m} \times \left\{1 - \frac{2[J_1(\sqrt{\alpha\varpi_m}) - C_m N_1(\sqrt{\alpha\varpi_m})]}{\sqrt{\alpha\varpi_m}[J_0(\sqrt{\alpha\varpi_m}) - C_m N_0(\sqrt{\alpha\varpi_m})]} - R^2\left[1 - \frac{2[J_1(R\sqrt{\alpha\varpi_m}) - C_m N_1(R\sqrt{\alpha\varpi_m})]}{R\sqrt{\alpha\varpi_m}[J_0(\sqrt{\alpha\varpi_m}) - C_m N_0(\sqrt{\alpha\varpi_m})]}\right]\right\}, \tag{25}$$

where, in this case, the dimensionless parameters are given by

$$\varpi_m = (\omega_m^*)^2 + i\omega_m^* - \frac{M^2}{\alpha}(1 - i\omega_m^*), \tag{26}$$

$$\omega_m^* = \omega t_m, \quad \alpha = \frac{\rho a^2}{\eta t_m} = \frac{t_v}{t_m},$$

with α^{-1} being the Deborah number. The parameter α determines whether the elastic or viscous behavior predominates. For the flow of Maxwell fluids in tubes in the absence of electromagnetic interaction, it was established that for values of α bigger than the critical value $\alpha_c = 11.64$, a dissipative behavior prevails, while for $\alpha < \alpha_c$ the prevalent elastic behavior leads to a resonance at a given frequency [6]. In this paper, we consider values of α much smaller than 1, so that elastic effects are intense though modulated by the magnetic interaction and the gap between the cylinders. Cases where $\alpha > 12$ are analogous to the Newtonian case analyzed in the previous section.

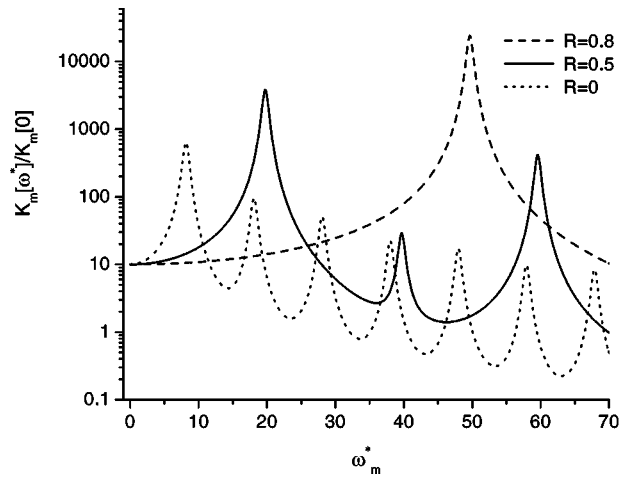

FIG. 3. Real part of the dimensionless dynamic permeability of a Maxwell fluid as a function of the normalized frequency parameter ω_m^* for different values of the dimensionless gap R. The magnetic field is absent and the Deborah number is $\alpha = 0.1$.

Similar to the Newtonian case, we normalize the dynamic permeability with the static permeability given by Eq. (25) evaluated at $\omega_m^* = 0$, i.e.,

$$K_m^*(\omega_m^*) = \frac{K_m(\omega)}{K_m(0)}. \tag{27}$$

Evidently, when the Maxwell relaxation time vanishes ($t_m \to 0$), Eq. (27) reduces to Eq. (15), i.e., $K_m^* \to K_n^*$. Another important limit is obtained when both $M \to 0$ and $R \to 0$. In such a case, we get the dynamic permeability reported in Ref. [7] for a viscoelastic fluid in a tube, namely,

$$\lim_{M,R\to 0} K_m^*(\omega_m^*) = -\frac{8(1 - i\omega_m^*)}{\alpha\varpi}\left\{1 - \frac{2J_1(\sqrt{\alpha\varpi})}{\sqrt{\alpha\varpi}J_0(\sqrt{\alpha\varpi_m})}\right\}, \tag{28}$$

where $\varpi = (\omega_m^*)^2 + i\omega_m^*$. Figure 3 shows the real part of the dimensionless dynamic permeability (27) as a function of the dimensionless frequency ω_m^* for $M = 0$, $\alpha = 0.1$, and different values of the gap parameter R. The case $R = 0$ calculated from Eq. (28), is also shown for comparison purposes. Since the magnetic interaction is absent, Fig. 3 displays essentially the effects of varying the space between the cylinders using a fluid with predominant elastic behavior. The appearance of multiple resonant frequencies, where the real part of the permeability increases, is observed. In all curves, the magnitude of the first peak is the highest and that of subsequent peaks steadily decays to zero, as in previous studied cases [6,7]. Comparing the curve $R = 0$ with those with $R \neq 0$, two main effects can be readily attributed to the presence of an inner coaxial cylinder. First, the shift of the frequencies that lead to the enhancement of the real part of the dynamic perme-

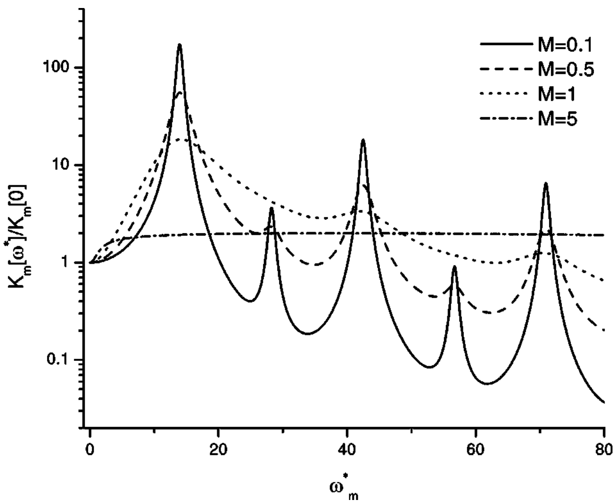

FIG. 4. Real part of the dimensionless dynamic permeability of a Maxwell fluid as a function of the normalized frequency parameter ω_m^* for different Hartmann numbers. The the dimensionless gap is $R=0.3$ and the Deborah number is $\alpha=0.1$.

ability to larger values, and second, the presence of resonant frequencies of small intensity between two higher peaks that exceed the maximum values of the permeability obtained for the case $R=0$. Here, the smaller the gap the larger the difference. Notice that the first maximum for the case $R=0.8$ is several orders of magnitude higher than the static value. This fact might be important for lubrication studies under cyclic regimes. Although not shown, corresponding imaginary parts of $K_m^*(\omega_m^*)$ present also a shift in the phase frequencies with respect to the single tube case [7].

Figure 4 displays the effect of the magnetic interaction on the dynamic permeability. It shows the real part of K_m^* as a function of ω_m^* for $R=0.3$, $\alpha=0.1$, and different values of the Hartmann number M. Clearly, the effect of the magnetic field is totally dissipative. As M grows from 0 to 1 the peaks of the dynamic permeability are attenuated. For $M=1$, only the first peak persists and $\mathrm{Re}\{K_m^*(\omega_m^*)\}$ decays monotonically to zero for large frequencies. Notice, however, that the rate of decrease of the permeability is smaller the larger the Hartmann number. In the limit of $M\to\infty$, whatever the value of α, the elastic behavior is completely inhibited by the Ohmic dissipation and $K_m^*\to 1$ for small ω_m^* and goes to zero when $\omega_m^*\to\infty$, as in the Newtonian case.

In the case of a single tube in the absence of a magnetic field, a simple relationship between the maximum permeability values and the Deborah number was found [7]. In the present case, however, the maximum value of $\mathrm{Re}\{K_m^*(\omega_m^*)\}$ depends on three independent parameters, namely, α, M, and R; therefore the determination of a relation between the maximum values of the permability and these parameters is not a simple matter and lies beyond the scope of the present investigation.

IV. CONCLUSION

In this paper, we have extended previous investigations on the dynamic permeability of flows in tubes by incorporating new geometric aspects and the existence of electromagnetic interaction. Specifically, we explored the dynamic behavior of the permeability of annular ducts saturated with electrically conducting fluids, either Newtonian or Maxwellian, under a constant radial magnetic field. In the absence of a magnetic field, the presence of the inner cylinder results advantageous for flows of Newtonian fluids in the sense that the range of frequencies, where the dynamic permeability values are close to the static value, is expanded. When the magnetic field is present, this frequency range is even wider. In fact, for high-magnetic field strengths the local acceleration is negligible and the permeability becomes that of a quasisteady flow within a very large range of frequencies. This fact could find application in the enhancement of lubrication systems using magnetic fields.

For Maxwellian fluids in the absence of a magnetic field, the inner cylinder modifies the spectrum of resonant frequencies and changes the monotonic decay of the permeability peaks. In fact, compared to the case of a single tube, the frequencies at which the dynamic permeability is enhanced, are shifted to larger values. Further, at some resonant frequencies, the maximum values of the permeability exceed those reached in the flow in a single tube [7], a result that, incidentally, may be of practial importance. It must be stressed that in the particular case of a zero mean perturbation, no enhancement of the dynamic permeability will be produced. However, whenever a nonzero mean periodic perturbation produces a net flow rate, the choice of a pressure pulse with the appropriate frequency may cause a dramatic enhancement on the dynamic permeability according to Eq. (27). On the other hand, the existence of magnetic field results is disadvantageous for the enhancement of the dynamic permeability. In this case, the dissipative behavior of the magnetic field dominates over the elastic effects and a drastic reduction of the enhanced dynamic permeability is observed. Nevertheless, a wide frequency range that leads to dynamic permeability values higher than the static value is still present.

ACKNOWLEDGMENTS

This research was partially supported by DGAPA-UNAM and CONACYT under projects IN103100 and G0044E, respectively. The authors thank Professor Mariano López de Haro for a critical reading of the manuscript and useful suggestions.

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